Q1

DISTANCE = MAGNITUDE

$$(2k+1)^2+(3k-3)^2+(13-14)^2=(\sqrt{163})^2$$

$$=163$$

SOLVE QUADRATIC

$$4k^2+4k+1+9k^2-18k+9+1-163=0$$

$$13k^2-14k-152=0$$

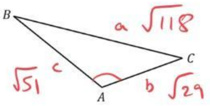
$$(13k+38)(k-4)=0$$

$$k=4 \qquad k=-\frac{38}{13}$$

$$\boxed{k=4}$$

Q2a

In the triangle ABC, $\overrightarrow{AB}=7\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\overrightarrow{AC}=-2\mathbf{i}+5\mathbf{k}$.



(a) Show that $\angle BAC = 119.6°$ to 1 d.p. [5]

COSINE RULE $\quad \cos A = \frac{b^2+c^2-a^2}{2bc}$

(b) Hence find the area of triangle ABC, giving your area to 3 s.f. [2]

a) $\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}$

$(-2\underline{i}+5\underline{k})-(7\underline{i}+\underline{j}-\underline{k})$

$-9\underline{i}-\underline{j}+6\underline{k}$

MAGNITUDE

$|AB|=\sqrt{7^2+1^2+1^2}=\sqrt{51}$

$|AC|=\sqrt{2^2+5^2}=\sqrt{29}$

$|BC|=\sqrt{9^2+1^2+6^2}=\sqrt{118}$

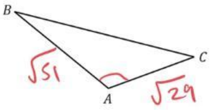
$$A=\cos^{-1}\left(\frac{29+51-118}{2\sqrt{29}\sqrt{51}}\right)$$

$$=119.6070\ldots$$

$$\boxed{\angle BAC=119.6 \text{ 1dp.}}$$

Q2b

In the triangle ABC, $\overrightarrow{AB} = 7\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\overrightarrow{AC} = -2\mathbf{i} + 5\mathbf{k}$.



(a) Show that $\angle BAC = 119.6°$ to 1 d.p.

(b) Hence find the area of triangle ABC, giving your area to 3 s.f.

$\text{AREA} = \frac{1}{2}ab\sin C$

b) $\text{AREA} = \frac{1}{2}\sqrt{51}\sqrt{29}\sin 119.6$

$= 16.7194\ldots$

$\boxed{16.7 \text{ UNITS}^2 \ (3sf)}$

Q3a

a) $\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR}$

$$\begin{pmatrix} 10 \\ 0 \\ 13 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ -6 \\ 15 \end{pmatrix}$$

$$\overrightarrow{RS} = 9\underline{i} - 6\underline{j} + 15\underline{k}$$

$$|RS| = \sqrt{9^2 + 6^2 + 15^2} = \sqrt{342}$$
$$= 3\sqrt{38}$$

$$\text{UNIT VECTOR} = \frac{\overset{3}{\cancel{9}}\underline{i} - \overset{2}{\cancel{6}}\underline{j} + \overset{5}{\cancel{15}}\underline{k}}{\cancel{3}\sqrt{38}}$$

$$\boxed{\begin{gathered} \frac{1}{\sqrt{38}}\left(3\underline{i} - 2\underline{j} + 5\underline{k}\right) \\ \text{OR} \\ \frac{3}{\sqrt{38}}\underline{i} - \frac{2}{\sqrt{38}}\underline{j} + \frac{5}{\sqrt{38}}\underline{k} \end{gathered}}$$

Q3b

b) ANGLE WITH Z AXIS $= \cos\theta_z = \frac{z}{|a|}$

$\overrightarrow{RS} = 9\underline{i} - 6\underline{j} + 15\underline{k}$

$|\overrightarrow{RS}| = 3\sqrt{38}$

$$\cos^{-1}\left(\frac{15}{3\sqrt{38}}\right) = 35.7957...$$

NEGATIVE $180 - \theta_z$

$$180 - 35.7957... = 144.20...$$

$$\boxed{144.2^\circ \text{ (1dp)}}$$

Q3C

c) PARALLEL = MULTIPLE

$\overrightarrow{RS} = 9i - 6j + 15k = \begin{pmatrix} 9 \\ -6 \\ 15 \end{pmatrix}$

$\overrightarrow{TU} = \begin{pmatrix} -12 \\ 8 \\ -20 \end{pmatrix}$ $\qquad \frac{\overrightarrow{TU}}{\overrightarrow{RS}} = -\frac{4}{3}$

$\overrightarrow{TU} = -\frac{4}{3}(\overrightarrow{RS})$

$\overrightarrow{TU}$ IS A MULTIPLE OF $\overrightarrow{RS}$ SO $\overrightarrow{TU}$ AND $\overrightarrow{RS}$ ARE PARALLEL

Q4

The triangle PQR has vertices $P(2,-3,1)$, $Q(-1,-4,3)$ and $R(k,0,3)$. Given that PQR is isosceles, and that $k>1$, find the value of k.

[4]

SIDE LENGTHS = MAGNITUDE

LEAVE IN SQUARE TERMS^2

$PQ^2 = (-3)^2 + (-1)^2 + 2^2 = 14$

$PR^2 = (k-2)^2 + 3^2 + 2^2 =$

$k^2 - 4k + 17$

$QR^2 = (k+1)^2 + 4^2$

$k^2 + 2k + 17$

ISOSCELES = TWO EQUAL SIDES

$PQ^2 = PR^2 \quad 14 = k^2 - 4k + 17$

$k^2 - 4k + 3 = 0$

$= (k-1)(k-3)$

$k=1 \quad k=3$

$PQ^2 = QR^2 \quad 14 = k^2 + 2k + 17$

$k^2 + 2k + 3 = 0$

NO SOLUTIONS

$PR^2 = QR^2$

$\cancel{k^2} - 4k + \cancel{17} = \cancel{k^2} + 2k + \cancel{17}$

$-4k = 2k \quad k=0$

$k > 1$

$\boxed{k=3}$

Q5

Vectors **a** and **b** are defined by

$$\mathbf{a} = (p+1)\mathbf{i} - 7\mathbf{j} + (q-3p)\mathbf{k}$$
$$\mathbf{b} = 5\mathbf{i} + (14q+r)\mathbf{j} + (1-2r)\mathbf{k}$$

Given that $\mathbf{a} = \mathbf{b}$, find the values of p, q and r.

[4]

EQUATE COEFFICIENTS

$(p+1)\underline{i} = 5\underline{i} \qquad p+1 = 5$

$p = 4$

$-7\underline{j} = (14q+r)\underline{j} \qquad -7 = 14q + r$ ①

$(q-3p)\underline{k} = (1-2r)\underline{k}$

$q - 12 = 1 - 2r$ ②

SOLVE SIMULTANEOUS EQUATIONS

$r = -14q - 7$

SUB INTO ②

$q - 12 = 1 - 2(-14q - 7)$

$q - 12 = 1 + 28q + 14$

$-27 = 27q$

$q = -1$

SUB INTO ①

$-7 = 14(-1) + r$

$-7 = -14 + r$

$r = 7$

$\boxed{p=4 \quad q=-1 \quad r=7}$

Q6a

a) $F = ma$ $\quad a = \frac{F}{m}$

$|a| = 26ms^{-2}$

$$a = \frac{12\underline{i} - 4\underline{j} + p\underline{k}}{0.5} = 24\underline{i} - 8\underline{j} + 2p\underline{k}$$

MAGNITUDE OF ACCELERATION

$$|a|^2 = 24^2 + 8^2 + (2p)^2 = 26^2$$

$$(2p)^2 = 36$$

$$2p = \sqrt{36} = \pm 6$$

$$p = \frac{\pm 6}{2} = \pm 3$$

$$\boxed{p = \pm 3}$$

Q6b

b) NEW ACCELERATION $8\sqrt{10} = 25.29...$

$$8\sqrt{10} < 26$$

NEW FORCE HAS REDUCED SIZE OF TOTAL FORCE SO AS $q < 0$ p MUST BE POSITIVE

Q6c

c) $p = 3$

RESULTANT

$$12\underline{i} - 4\underline{j} + (3+q)\underline{k}$$

$\div 0.5$

$a = \frac{F}{m}$

$$24\underline{i} - 8\underline{j} + 2(3+q)\underline{k}$$

NEW FORCE MAGNITUDE OF ACCELERATION

$$|a|^2 = \cancel{24^2 + 8^2} + (2(3+q))^2 = \cancel{(8\sqrt{10})^2}$$

$$(2(3+q))^2 = 0$$

$$3 + q = 0$$

$$\boxed{q = -3}$$

Q7a

Three forces act upon a particle with a mass of 5 kg:

$\mathbf{F}_1 = (r\mathbf{i} + 5\mathbf{j} + (r-p)\mathbf{k})$ N

$\mathbf{F}_2 = ((p+q)\mathbf{i} - 3p\mathbf{j} + 7\mathbf{k})$ N

$\mathbf{F}_3 = (-\mathbf{i} + r\mathbf{j} - 2q\mathbf{k})$ N

Under the action of those three forces, the particle is in equilibrium.

(a) Find the values of p, q and r. [4]

A new force, $\mathbf{F}_4$, is added. Under the combined action of the four forces, the particle experiences an acceleration with a magnitude of $2.2\ \mathrm{m\,s^{-2}}$, in the same direction as the vector $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

(b) Find $\mathbf{F}_4$, giving your answer in the form $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ where x, y and z are given as exact values. Be sure to include the correct units in your answer. [5]

a) EQUILIBRIUM $\Rightarrow$ RESULTANT VECTOR $= 0$

$\underline{i} = r + p + q - 1 = 0$

$r + p + q = 1$ ①

$\underline{j} = 5 - 3p + r = 0$

$r - 3p = -5$ ②

$\underline{k} = (r - p) + 7 - 2q = 0$

$r - p - 2q = -7$ ③

SOLVE SIMULTANEOUS EQUATIONS

③ + 2 × ① $\quad r - p - \cancel{2q} = -7$

④ $\quad 2r + 2p + \cancel{2q} = 2$

$\cancel{3r} + p = -5$

$-3$② $\quad \cancel{3r} - 9p = -15$

$10p = 10$

$p = 1$

SUB INTO ②

$r - 3 = -5$

$r = -2$

SUB INTO ①

$-2 + 1 + q = 1$

$q = 2$

$$\boxed{p = 1 \qquad q = 2 \qquad r = -2}$$

Q7b

b) $\underline{F_1} + \underline{F_2} + \underline{F_3} = 0$

$0 + \underline{F_4} = \underline{F_4}$ $\quad F = ma$

$\underline{a}_4 = 2.2 \times$ UNIT VECTOR $\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

MAGNITUDE

$\sqrt{1^2 + 1^2 + 3^2} = \sqrt{11}$

$\underline{a} = 2.2 \times \frac{1}{\sqrt{11}} \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

$F_4 = 5 \times \frac{2.2}{\sqrt{11}} \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \frac{11}{\sqrt{11}} \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

RATIONALISE $\frac{11}{\sqrt{11}} = \sqrt{11}$

$$\boxed{\sqrt{11}\,\underline{i} - \sqrt{11}\,\underline{j} + 3\sqrt{11}\,\underline{k}\ \text{N}}$$

Q8

FIND $\overrightarrow{CD}$ AND $\overrightarrow{BF}$ [9]

$\overrightarrow{CD} = \underline{a} + \underline{b} - \underline{c} \qquad \overrightarrow{BF} = \underline{a} - \underline{b} + \underline{c}$

SETUP POSITION VECTORS $\overrightarrow{OP}$ AND $\overrightarrow{OQ}$

FOR A POINT P ON $\overrightarrow{CD}$ AND A POINT Q ON $\overrightarrow{BF}$

$\overrightarrow{OP} = \underline{c} + \lambda \overrightarrow{CD} \qquad \overrightarrow{OQ} = \underline{b} + \mu \overrightarrow{BF}$

EQUATE AND SOLVE

IF $\vec{OP} = \vec{OQ}$

$$\underline{c} + \lambda \vec{CD} = \underline{b} + \mu \vec{BF}$$

$$\underline{c} + \lambda(\underline{a} + \underline{b} - \underline{c}) = \underline{b} + \mu(\underline{a} - \underline{b} + \underline{c})$$

$$\lambda \underline{a} + \lambda \underline{b} + (1-\lambda)\underline{c} = \mu \underline{a} + (1-\mu)\underline{b} + \mu \underline{c}$$

$$\lambda = \mu \qquad \lambda = 1 - \mu \qquad 1 - \lambda = \mu$$

SUB TO SOLVE

$$\mu = 1 - \mu \qquad \lambda = 1 - \lambda$$

$$2\mu = 1 \qquad 2\lambda = 1$$

$$\mu = \frac{1}{2} \qquad \lambda = \frac{1}{2}$$

$\vec{CD}$ AND $\vec{BF}$ INTERSECT AT

$$\frac{1}{2}\underline{a} + \frac{1}{2}\underline{b} + \frac{1}{2}\underline{c}$$

BOTH μ AND λ ARE $\frac{1}{2}$ MEANING $\vec{CD}$ AND $\vec{BF}$ BISECT EACHOTHER